

# JHU CLSP Workshop 2009

## **Motivating Sub-Space Modeling for Automatic Speech Recognition**

# OUTLINE

- Introduce the Notion of Identifying Low Dimensional Subspaces over Model Parameters
- Subspace Based Speaker Adaptation
- Generalization to a Generalized Joint Subspace Model of Acoustic Variability in ASR

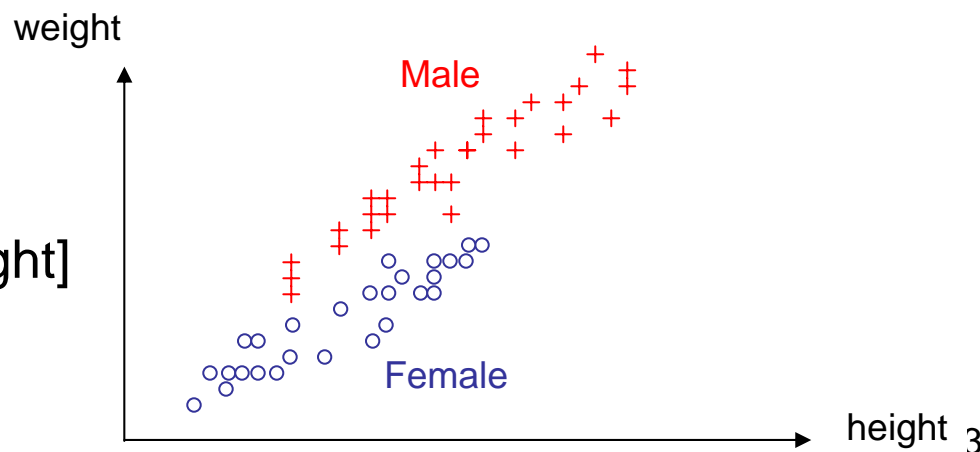
# Identifying Low Dimensional Feature Space – Dimensionality Reduction

- Problems with high dimensional *feature spaces*:
  - High computational complexity
  - Poor generalization to unseen data
  - “Curse of dimensionality”
- Starting with high dimensional *data*, identify low dimensional feature space
  - Principal components analysis (PCA) – Capture maximum variance
  - Linear Discriminant Analysis (LDA) – Maximum class separability

Example:

2-D Feature Space: [height, weight]

2 Classes: **male**, **female**

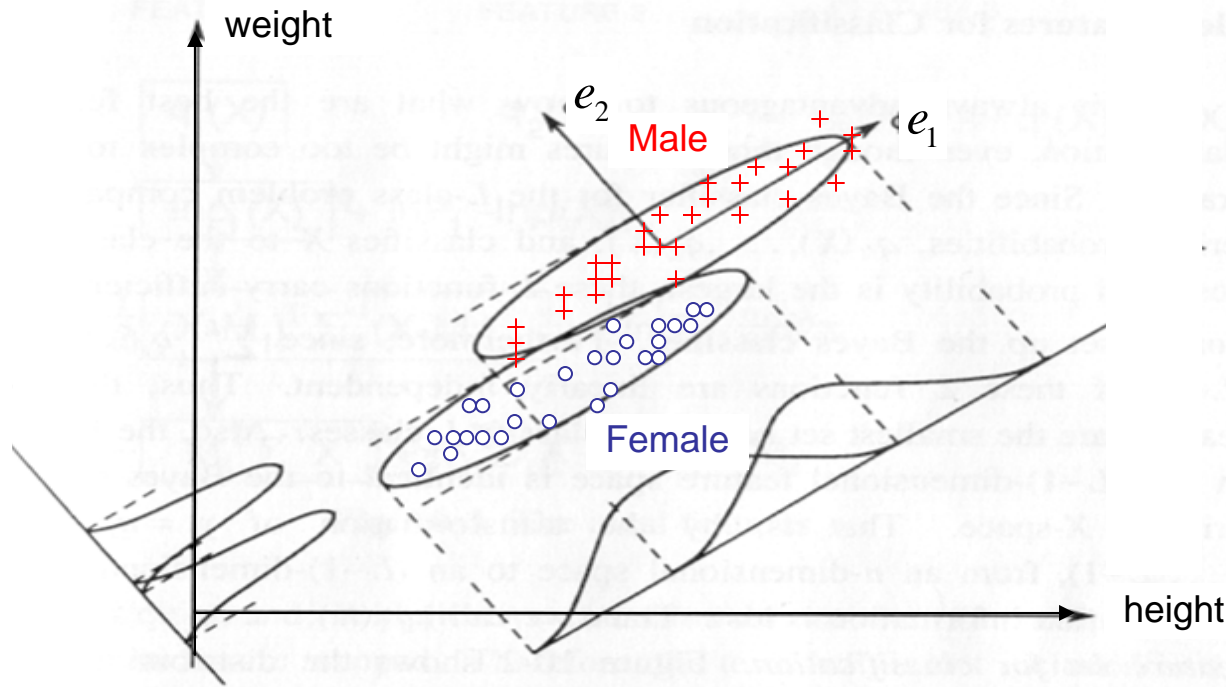


# Identifying Low Dimensional Feature Space

- Low dimensional feature,  $y$ , obtained from high dimensional feature,  $x$ , by a linear transformation:

Maximum Separability:  $y = e_2^T x$   
 (Linear Discriminant Analysis)

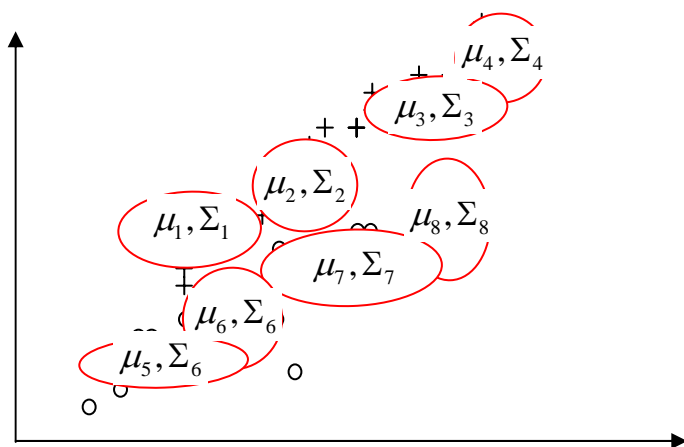
Maximum Variance:  $y = e_1^T x$   
 (Principal Components Analysis)



# Identifying Low Dimensional Model Space

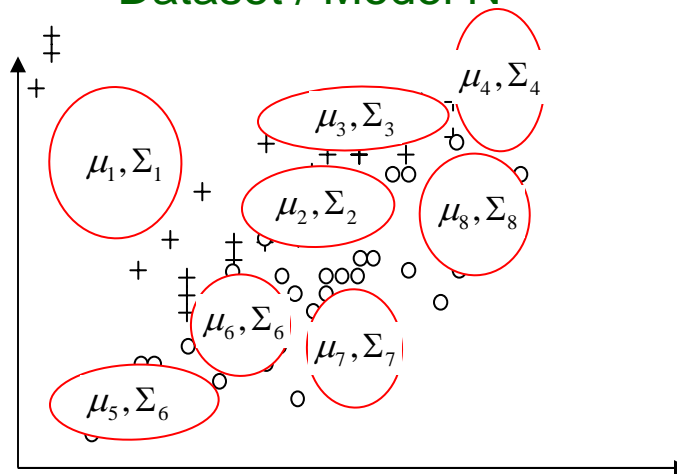
- Suppose there are multiple data sets describing similar populations and models are to fit each data set:

Dataset / Model 1



...

Dataset / Model N



- A low dimensional subspace can be identified that describes variation of the parameters

Form Super Vectors from Models:

$$\boldsymbol{\mu}^1 = \begin{bmatrix} \mu_1^1 \\ \vdots \\ \mu_8^1 \end{bmatrix}, \dots, \boldsymbol{\mu}^N = \begin{bmatrix} \mu_1^N \\ \vdots \\ \mu_8^N \end{bmatrix}$$

Estimate Sub-space Projection:

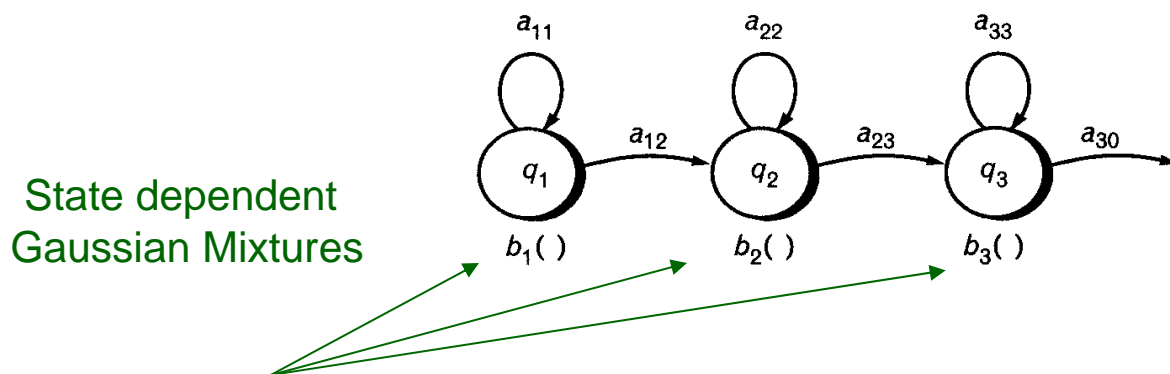
$$\boldsymbol{\mu} = \mathbf{E}\mathbf{v}$$

... or: 
$$\boldsymbol{\mu} = \mathbf{m}_0 + \mathbf{E}\mathbf{v}$$



# Speaker Space Adaptation – Super-vector

- Continuous Gaussian Mixture Observation Density HMMs



$$p(\vec{x} | s_j) = \sum_{m=1}^{C_j} w_{m,j} f_{m,j}(\vec{x}) , \text{ with } C = \sum_m C_m \text{ mixture components, and } f_{m,j}(\vec{x}) : N[\vec{x}; \vec{\mu}_{m,j}, \Sigma_{m,j}]$$

- A speaker,  $S$ , is generally defined over a “super-vector” of the concatenated means of component Gaussians:

$$\vec{\mu}^s = \begin{bmatrix} \mu_1^s \\ \mu_2^s \\ \vdots \\ \mu_C^s \end{bmatrix} \left. \vphantom{\begin{bmatrix} \mu_1^s \\ \mu_2^s \\ \vdots \\ \mu_C^s \end{bmatrix}} \right\} \begin{array}{l} \text{Dimension:} \\ M = CF \end{array}$$

- Example: Wall Street Journal HMM
  - Component Gaussians  $C \approx 100,000$
  - Feature Vector Dimension  $F \approx 40$
  - Super Vector Dimension  $CF \approx 4,000,000$
- Super-vector dimension can be very large

# Speaker Space Based Adaptation

- Adapt Super-vector in Low Dimensional Subspace
- **Training (Off-Line):** Identify basis vectors of low dimensional speaker subspace from speaker dependent super-vectors:

$$\vec{\mu}^1, \dots, \vec{\mu}^S \longrightarrow \mathbf{E} = \vec{e}^1, \dots, \vec{e}^K$$

where  $\vec{\mu}^s$  is dimension,  $M$ , and  $\mathbf{E}$  is dimension  $M \times K$  where  $K \ll M$

- **Adaptation:** Estimate weights  $w_k^s, k = 1, \dots, K$  from adaptation data to obtain adapted super-vector:

$$\hat{\vec{\mu}}^s = \vec{\mu}^{SI} + \sum_{k=1}^K w_k^s \vec{e}(k)$$

- Requires only a few seconds of adaptation data
- Speaker subspace dimension  $K \approx 10 \rightarrow 100$

# Subspace Identification – Training

- Principal Components Analysis (EigenVoices)

- Starting from M dimensional super-vectors for each of S speakers to a K dimensional subspace

$$\begin{array}{c|c|c} \bar{\mu}_1^1 & \cdots & \bar{\mu}_1^S \\ \vdots & \ddots & \vdots \\ \bar{\mu}_C^1 & \cdots & \bar{\mu}_C^S \end{array} \xrightarrow[\substack{\text{PCA} \\ K \ll CF}]{\text{}} \begin{array}{c|c|c} \bar{e}(1,1) & \cdots & \bar{e}(1,K) \\ \vdots & \ddots & \vdots \\ \bar{e}(M,1) & \cdots & \bar{e}(M,K) \end{array}$$

Speaker Super-vectors for  
Speaker 1 through Speaker S

Speaker Subspace  
basis vectors

- Maximum Likelihood Clustering (Cluster Adapt. Training)

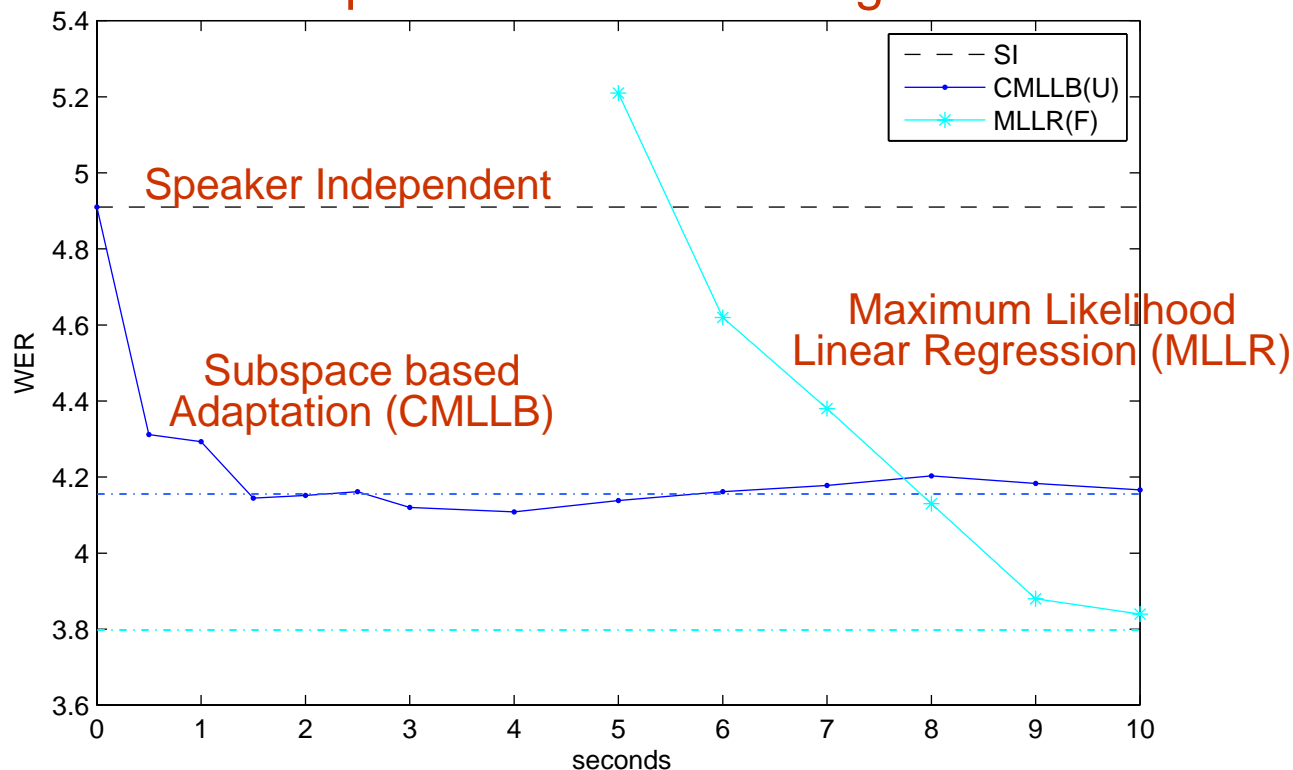
- Given SD training observation vectors,  $X^s = x_1^s, \dots, x_T^s$ , SI HMM model,  $\lambda^{SI}$ , and initial estimate of  $\mathbf{E}^{(0)} = \bar{\mathbf{e}}^1, \dots, \bar{\mathbf{e}}^K$
- Use EM algorithm to iteratively estimate weights and basis vectors

$$\hat{\Lambda}: \hat{\mu}^s = \bar{\mu}^{SI} + \sum_{k=1}^K w_k^s \bar{e}(k)$$



# Limited Effect of “Global” Subspace Adaptation

WER vs. Adaptation Utterance Length on RM Task



[From Tang and Rose, 2008]

- Substantial improvement with only 1 or 2 seconds of adaptation data
- Does not exhibit desirable asymptotic behavior

# Generalization to Subspace Models of Phonetic Variability

- Speaker space model is limited in the form of the variability it can represent
  - Single vector in speaker space describes speaker specific variability
- Generalize this model in three ways:
  1. Define **multiple model subspaces** over different regions of the feature space
  2. Define **state-specific**, (rather than speaker specific) **weight vectors** to describe phonetic variation within these subspaces
  3. Define **joint model / speaker subspaces**
- This is conceptually a straightforward generalization of the speaker subspace approach

# Generalization of Sub-Space HMM

- Review: Subspace Based Speaker Adaptation**

- Single global subspace,  $\mathbf{N}$ , defined over all Gaussians in HMM
- Single weight vector,  $v^{(s)}$ , describes variation in subspace

$$\hat{\mu}_{j,m}^{(s)} = \mu_{j,m}^{SI} + \mathbf{N}v^{(s)}$$

$$p(x | s_j) = \sum_{m=1}^M w_{j,m} p(x; \hat{\mu}_{j,m}^{(s)}, \Sigma_{j,m})$$

- Generalization: Multiple Region-Specific Subspaces**

- Separate Subspaces,  $\mathbf{N}_i$ , defined for each Gaussian in a GMM
- Single weight vector describes variation in subspaces

$$\hat{\mu}_i^{(s)} = \mu_i^{UBM} + \mathbf{N}_i v^{(s)}$$

$$p(x | s_j) = \sum_{i=1}^I w_{j,i} p(x; \hat{\mu}_i^{(s)}, \Sigma_i)$$

## Generalization to Joint Subspace HMM

- **Generalization: State-Specific Weight Vectors**
  - Subspaces,  $\mathbf{M}_i$ , defined over shared pool of Gaussians
  - State-specific weight vectors,  $\mathbf{v}_j$ , describe phonetic var. in subspace

$$\hat{\mu}_{j,i} = \mathbf{M}_i \mathbf{v}_j \quad p(x | s_j) = \sum_{i=1}^I w_{j,i} p(x; \hat{\mu}_{j,i}, \Sigma_i)$$

- **Generalization: Joint Model / Speaker Subspaces**
  - Model and Speaker subspaces,  $\mathbf{M}_i$  and  $\mathbf{N}_i$
  - State-specific and speaker specific weight vectors  $\mathbf{v}_j$  and  $\mathbf{v}^{(s)}$

$$\hat{\mu}_{j,i}^{(s)} = \mathbf{M}_i \mathbf{v}_j + \mathbf{N}_i \mathbf{v}^{(s)}$$

# Subspace HMM

Observation Prob.  
for State  $j$ :

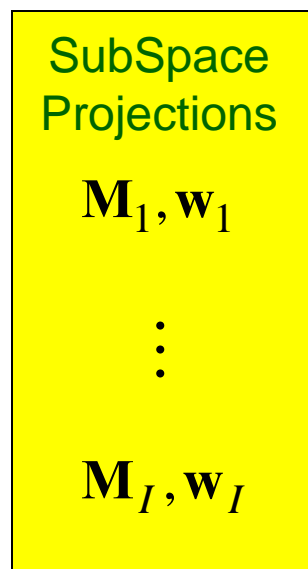
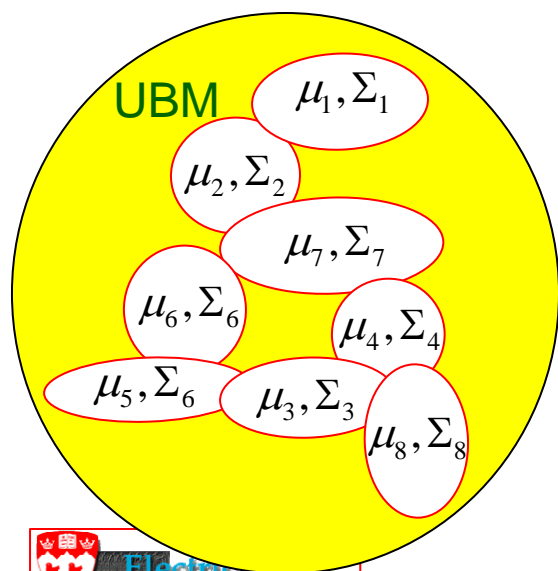
$$p(x | s_j) = \sum_{i=1}^I w_{j,i} p(x; \mu_{j,i}, \Sigma_i)$$

Projection of State-Specific Vectors:

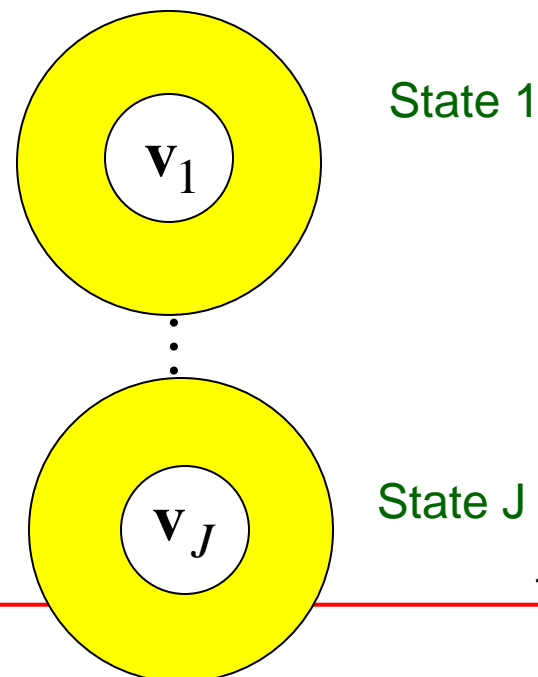
$$\mu_{j,i} = \mathbf{M}_i \mathbf{v}_j$$

$$w_{j,i} = \frac{\exp \mathbf{w}_i^T \mathbf{v}_j}{\sum_{l=1}^I \exp \mathbf{w}_l^T \mathbf{v}_j}$$

Shared Parameters



State Specific Parameters



# Subspace HMM – Multiple Substates

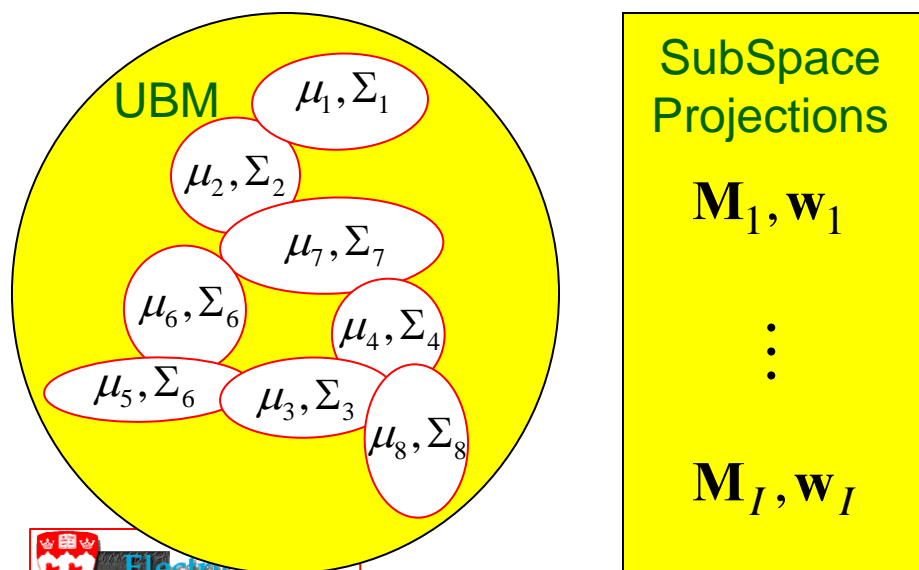
Observation Prob.  
for State  $j$ :

$$p(x | s_j) = \sum_{m=1}^M c_{j,m} \sum_{i=1}^I w_{j,m,i} P(x; \mu_{j,m,i}, \Sigma_i)$$

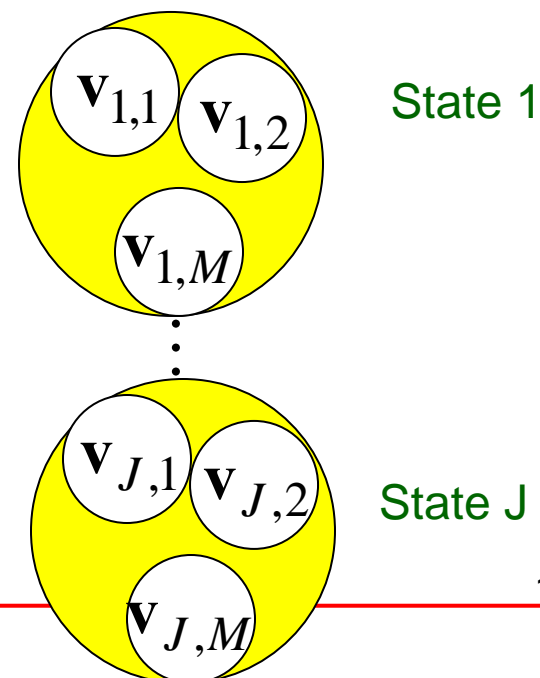
Projection of State-Specific Vectors:

$$\mu_{j,m,i} = \mathbf{M}_i \mathbf{v}_{j,m} \quad w_{j,m,i} = \frac{\exp \mathbf{w}_i^T \mathbf{v}_{j,m}}{\sum_{l=1}^I \exp \mathbf{w}_l^T \mathbf{v}_{j,m}}$$

Shared Parameters



State / Sub-state Specific Parameters



## Empirical Trade-Off: Shared and State-Specific Parameters

- Example: Wall Street Journal HMM model:
- 6000 states, 100,000 Gaussians  $\Rightarrow$  ***~8 Million parameters***
- Possible Substate HMM Parameterizations:

Parameter Allocation			Number of parameters		
UBM Gaussians	Sub-space Dim.	Sub-States	Shared	State-Specific	Total
256	39	1	600K	235K	835K
1024	39	1	2.4M	235K	2.65M
1024	100	1	4.8M	600K	5.4M
256	39	16	600K	3.7M	4.3M

# Summary

- The workshop is investigating a subspace based alternative to HMM models that includes:
  1. **multiple model subspaces** defined over different regions of the feature space
  2. **state-specific weight vectors** for describing phonetic variation within these subspaces
  3. **joint model / speaker subspaces**
- Workshop goals are to investigate:
  - Potential for sharing training data across languages and task domains
  - Empirical trade-off between number of Gaussians, states, sub-states, and sub-state dimension
  - Effects of joint subspaces: modeling both phonetic and speaker variation